Various positions on testing Inequality constrained hypotheses for LTA results

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In this paper we briefly explain how results from Latent Transition Analyses (LTA) can be evaluated by evaluating inequality constrained hypotheses. We describe two methods to test for such hypotheses and we explain that for one method the assumption of independence is violated. We introduce an alternative method where the assumption is not violated. As a case study we use the analyses we ran for a paper on comorbidity of aggression and anxiety in adolescence.

Latent Transition Analysis

As described in Meeus et al. (2010), "latent transition analysis represents a longitudinal extension of LCA […]. LTA calculates patterns of stability and change over time in the form of movement or transitions between classes (identity statuses in this case). Like LCA, LTA models use class-specific parameters (the continuous scores for each of the identity variables within each class) as measurement parameters, and class probabilities as structural parameters to estimate the number of participants in each of the classes. To model change over time, LTA adds a second set of structural parameters, latent transition probabilities, to the latent class model. In a two-wave LTA, for example, transition probabilities refer to the probability of moving into class Y in Wave 2 conditional on having been in class X in Wave 1. These transition probabilities range between 0 and 1. In sum, then, LTA offers two types of structural parameters: (a) varying numbers of participants in class across waves, indicating increase or decrease in class size over time, and (b) transitions of individuals between classes that carry these changes of class size." (p. 1570).

Latent transition analysis results can be converted into contingency tables summarizing the prevalence of classes across waves. In the paper of Meeus, van de Schoot, Hawk, Hale, and Branje (2013), using LTA, four latent groups of comorbidity types of aggression and anxiety in adolescents were found: anxious, aggressive, comorbid aggressive adolescents, and adolescents with no problems. With 5 waves of data, the resulting contingency table is shown in Table 1, where only the information of wave 1 and wave 5 has been used.

Table 1

Number and Percentage of Comorbidity Types by Wace (N =1313)

	Comorbidity type							
	Anxious		Aggressive		Comorbid aggressive		No problems	
Wave	*N*	%	*N*	%	*n*	%	*n*	%
1	182	13.9	322	24.5	102	7.8	707	53.8
5	207	15.8	308	23.5	58	4.4	740	56.4

Inequality Constrained Hypotheses

If researchers have specific hypothesis about contingency tables Bayesian model selection can be used to test (in)equality constraints between the parameters of interest (Hoijtink, 2012; Klugkist, Laudy & Hoijtink, 2010). As was explained in, for example, Wong and Van de Schoot (2012): "Consider the [3 x 3] contingency table presented in Table [2], with two variables (X and Y). In this table π_{ij} denotes the cell probabilities in the contingency table based on the LTA output; *i* denote the categories of the row variable, *j* denote the categories of the column variable. The association between Groups and Occasion can for example be evaluated in terms of conditional probabilities. For example $\pi_{11} / (\pi_{11+} \pi_{21+} \pi_{31}) > \pi_{21} / (\pi_{11+} \pi_{21+} \pi_{31})$, where the probabilities are conditioned on the row totals and '>' indicates that the conditional probability in cell *11* is larger than in cell *21*." (p.1281).

Table 2. *A hypothetical 3 X 3 contingency table.*

		X		
		1	2	3
Y		π_{11}	π_{21}	π_{31}
		π_{12}	π_{22}	π_{32}
		π_{13}	π_{23}	π_{33}

In Meeus et al (2013) one (out of many) of the hypothesis was: "to test which of three alternative models of increase and decrease of comorbidity types best fit the data. Model 1 assumed no increase or decrease of comorbidity types across waves 1 and 5, whereas Model 2 assumed an increase of the anxious and no problems type and a decrease of both aggressive types. In Model 3, the unconstrained model, the distribution of comorbidity types over time was allowed to vary freely; no constraints were specified between the comorbidity types across waves 1 and 5, thereby assuming that every cell size was equally likely."

Two methods to evaluate inequality constraints for LTA results

To test the hypotheses about the increase/decrease over time there are two methods available.

Method 1

The first method which could be used to evaluate the hypotheses of Meeus et al. (2013) is to translate the information in Table 1 into an inequality constrained hypothesis using the coding scheme as displayed in Table 3.

Table 3. *Inequality constrained coding scheme*

Wave:	Anxious	Aggressive	Comorbid aggressive	No Problems
1	P1	P2	P3	P4
5	P5	P6	P7	P8

The expectations of Meeus et al can now be formulated as an inequality constrained hypotheses:

Model 1: No increase or decrease of the 4 comorbidity types

"(p1/(p1+p2+p3+p4))=(p5/(p5+p6+p7+p8))"
"(p2/(p1+p2+p3+p4))=(p6/(p5+p6+p7+p8))"
"(p3/(p1+p2+p3+p4))=(p7/(p5+p6+p7+p8))"
"(p4/(p1+p2+p3+p4))=(p8/(p5+p6+p7+p8))"

Model 2: Increase of anxious and no problems and decrease of aggressive and aggressive comorbid

"(p5/(p5+p6+p7+p8))>(p1/(p1+p2+p3+p4))"
"(p2/(p1+p2+p3+p4))>(p6/(p5+p6+p7+p8))"
"(p3/(p1+p2+p3+p4))>(p7/(p5+p6+p7+p8))"
"(p8/(p5+p6+p7+p8))>(p4/(p1+p2+p3+p4))"

Model 3: Unconstrained

The results of this model are in terms of Bayes Factors (BFs):

$BF_{13} < .001$
$BF_{23} = 6.55$
$BF_{12} = 6.55 / .001 = 6{,}550.$

And in terms of posterior model probabilities (PMPs) which are standardized BFs:

$PMP_1 < .001$
$PMP_2 = .99$
$PMP_3 < .001.$

However, when using the method described above, the assumption of independence is violated. That is, the first row in Table 1 is not independent from the second row. That is, the same respondent is part of the first row *and* the second row. This affects the results because the tests are computed on more data than actually is present in the data set. When analysing the data for the first time, we were unaware of this assumption. As was correctly noted by dr. I. Klugkist (personal communication. 2013) in Method 1 the model specification was incorrect. Therefore, the BF values as reported above are biased. Because, artificially, more data has been used than should have been used, the reported BF is overestimated.

Method 2

Upon the information by dr. Klugkist we re-analysed the data of Meeus et al. (2013). In Table 4 we have re-organized the contingency table of Table 1 in such a way that the assumption of independence is not violated and hypotheses about the increase/decrease over time can be tested.

Table 4. *Alternative way of organizing the contingency table of Table 1.*

		Wave 5			
		Anxious	Aggressive	Comorbid aggressive	No problems
Wave 1	Anxious	128	22	3	29
	Aggressive	16	159	30	117
	Comorbid aggressive	12	51	14	25
	No problems	51	76	11	569

Table 5. *Inequality constrained coding scheme*

		Wave 5			
		Anxious	Aggressive	Comorbid aggressive	No problems
Wave 1	Anxious	P1	P2	P3	P4
	Aggressive	P5	P6	P7	P8
	Comorbid aggressive	P9	P10	P11	P12
	No problems	P13	P14	P15	P16

Using the coding scheme in Table 5 the correct model specifications to test the expectations of Meeus et al. are:

Model 1: No increase or decrease of the 4 comorbidity types

"((p1+p2+p3+p4)/(p1+p2+p3+p4+p5+p6+p7+p8+p9+p10+p11+p12+p13+p14+p15+p16))=
((p1+p5+p9+p13)/(p1+p2+p3+p4+p5+p6+p7+p8+p9+p10+p11+p12+p13+p14+p15+p16))"

"((p5+p6+p7+p8)/(p1+p2+p3+p4+p5+p6+p7+p8+p9+p10+p11+p12+p13+p14+p15+p16))=
((p2+p6+p10+p14)/(p1+p2+p3+p4+p5+p6+p7+p8+p9+p10+p11+p12+p13+p14+p15+p16))"

"((p9+p10+p11+p12)/(p1+p2+p3+p4+p5+p6+p7+p8+p9+p10+p11+p12+p13+p14+p15+p16))=
((p3+p7+p11+p15)/(p1+p2+p3+p4+p5+p6+p7+p8+p9+p10+p11+p12+p13+p14+p15+p16))"

"((p4+p8+p12+p16)/(p1+p2+p3+p4+p5+p6+p7+p8+p9+p10+p11+p12+p13+p14+p15+p16))=
((p13+p14+p15+p16)/(p1+p2+p3+p4+p5+p6+p7+p8+p9+p10+p11+p12+p13+p14+p15+p16))"

Model 2: Increase of anxious and no problems and decrease of aggressive and aggressive comorbid

"((p1+p5+p9+p13)/(p1+p2+p3+p4+p5+p6+p7+p8+p9+p10+p11+p12+p13+p14+p15+p16))>
((p1+p2+p3+p4)/(p1+p2+p3+p4+p5+p6+p7+p8+p9+p10+p11+p12+p13+p14+p15+p16))"

"((p5+p6+p7+p8)/(p1+p2+p3+p4+p5+p6+p7+p8+p9+p10+p11+p12+p13+p14+p15+p16))>
((p2+p6+p10+p14)/(p1+p2+p3+p4+p5+p6+p7+p8+p9+p10+p11+p12+p13+p14+p15+p16))"

"((p9+p10+p11+p12)/(p1+p2+p3+p4+p5+p6+p7+p8+p9+p10+p11+p12+p13+p14+p15+p16))>
((p3+p7+p11+p15)/(p1+p2+p3+p4+p5+p6+p7+p8+p9+p10+p11+p12+p13+p14+p15+p16))"

"((p4+p8+p12+p16)/(p1+p2+p3+p4+p5+p6+p7+p8+p9+p10+p11+p12+p13+p14+p15+p16))>
((p13+p14+p15+p16)/(p1+p2+p3+p4+p5+p6+p7+p8+p9+p10+p11+p12+p13+p14+p15+p16))"

Model 3: Unconstrained.

The new results in terms of BFs are:

BF_{13}= .0005;

BF_{23}= 7.30;

BF_{12}= 7.30 / .005 = 1,460.

And in terms of posterior model probabilities (PMPs) which are standardized BFs:

PMP_1<.001

PMP_2=.879

PMP_3=.120

Conclusion

In conclusion, the BFs/PMPs from method 2 are only slightly different form method 1, but the main conclusion does not differ.

After completion of the Meeus et al (2013) paper we checked the analyses of two other papers that used also LTA: Meeus, Van de Schoot**,** Keijsers, Schwartz, and Branje (2010), and Meeus, van de Schoot, Klimstra, and Branje (2011). We reanalysed the data of one research question and one hypothesis. For RQ1 from Meeus et al. (2010) the recalculated BF's were: $BF_{2,1}$ = 15,514 and $BF_{2,unc}$ = 21.41, and for hypothesis 1.1 from Meeus et al. (2011) $BF_{2,1}$ = 5979.90 and $BF_{2,unc}$ = 5.97, respectively. The PMP's were very similar to the values reported in the papers. For RQ1 from Meeus

et al. (2010) the reported values were PMP_1=.03, PMP_2=.97, and PMP_3<.001, respectively, and the recalculated values were PMP_1<.001, PMP_2=.95, and PMP_3<.05, respectively. For hypothesis 1.1 from Meeus et al. (2011), the values of the reported and the recalculated PMP's were exactly the same: PMP_1<.001, PMP_2=.99, and PMP_3<.001, respectively.

Therefore, we conclude that the original conclusions of Meeus and colleagues (2010, 2011) are robust. Of course, we do recommend the second method for further studies, as done in Meeus et al (2013).

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